Quasi-normal modes and non-linearities

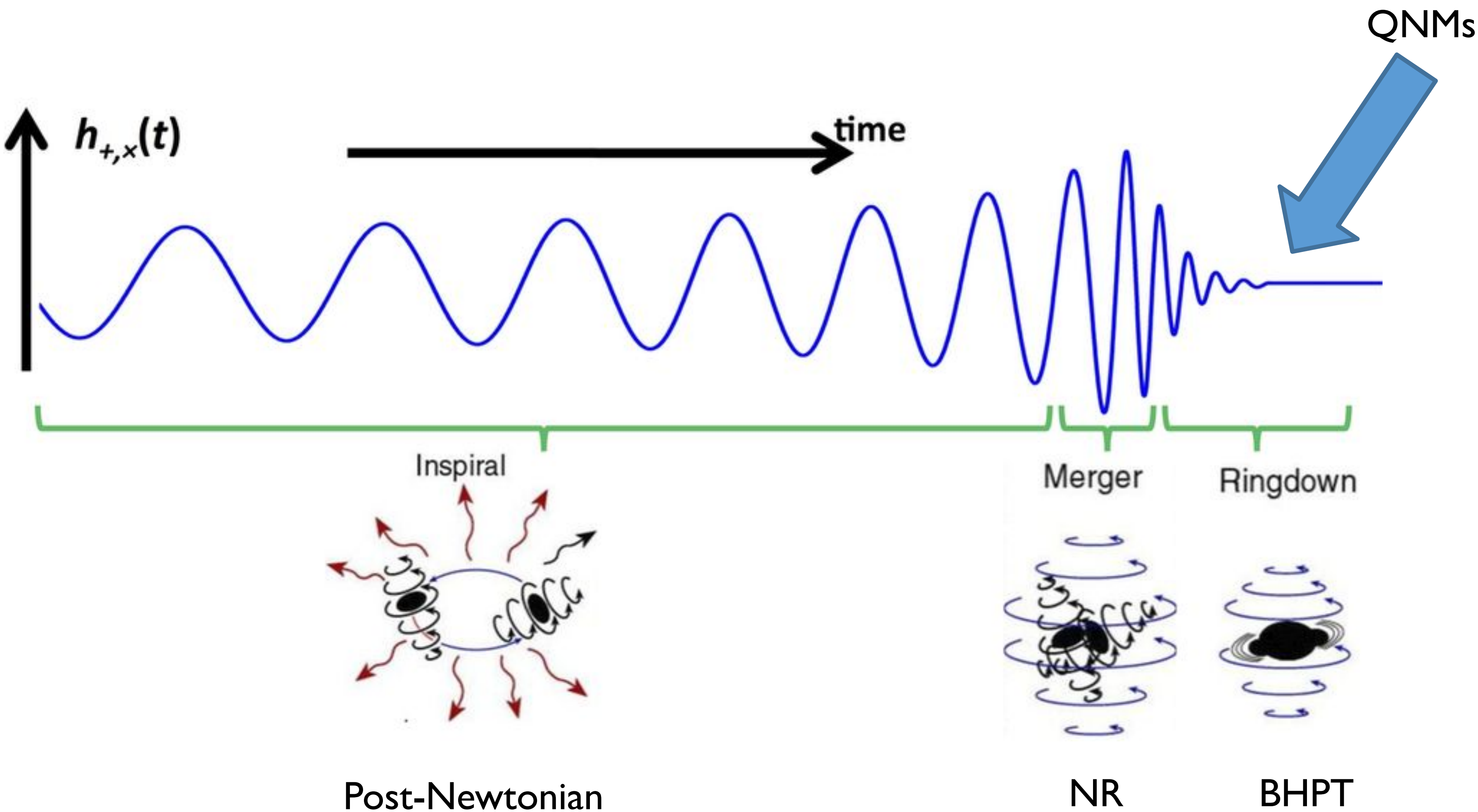
Béatrice Bonga - 22 March 2024

[Neev Khera, Ariadna Ribes Metidieri, BB, Xisco Jiménez Forteza, Badri Krishnan, Eric Poisson, Daniel Pook-Kolb, Erik Schnetter, Huan Yang

PRL, arXiv:2306/11142]

Radboud University

Gravitational waves from black hole mergers



Quasi-normal modes

Mathematical description

$$h_+ + i\,h_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{lm}(t,r) \;\; {}_2Y_{lm}(\theta,\phi)$$

spin-weighted
spherical harmonic

$$h_{lm}(t,r) = \frac{1}{r} \sum_{n=0}^{N} A_{lmn} e^{-i\omega_{lmn}(t-t_0)+\Phi_{lmn}}$$

Depend on the details
of the “hammer”

Frequencies and damping times

$$\omega_{lmn} = \omega^R_{lmn} + i\,\omega^I_{lmn} = 2\pi f_{lmn} + \frac{i}{\tau_{lmn}}$$

Depends on three integers:

$$l = 2,3,\ldots$$
$$-l < m < l$$
$$n = 0,1,2,\ldots$$

Damping time

Frequencies can be calculated using black hole perturbation theory

Non-linearities?

Perturbation theory

$$\Psi^{(1)} \sim A^{(1)}_{\pm,lmn}(r)e^{-i\omega_{\pm,lmn}t+i\phi_{\pm,lmn}}\,{}_2Y_{lm}(\theta,\varphi)$$

$$\mathcal{O}\Psi^{(1)} = 0$$

$$\mathcal{O}\Psi^{(2)} = \mathcal{S}(h^{(1)}, h^{(1)})$$

$$\Psi^{(2)} \sim A^{(2)}_{\pm,lmn}(r)e_2^{-i\omega^{(2)}_{\pm,lmn}t+i\phi_{\pm,lmn}}Y_{lm}(\theta,\varphi)$$

$$\omega_{lmn\times l'm'n'} = \omega_{lmn} + \omega_{l'm'n'}$$

Amplitude relation

$$\mathcal{O}\Psi^{(2)} = \mathcal{S}(h^{(1)}, h^{(1)})$$

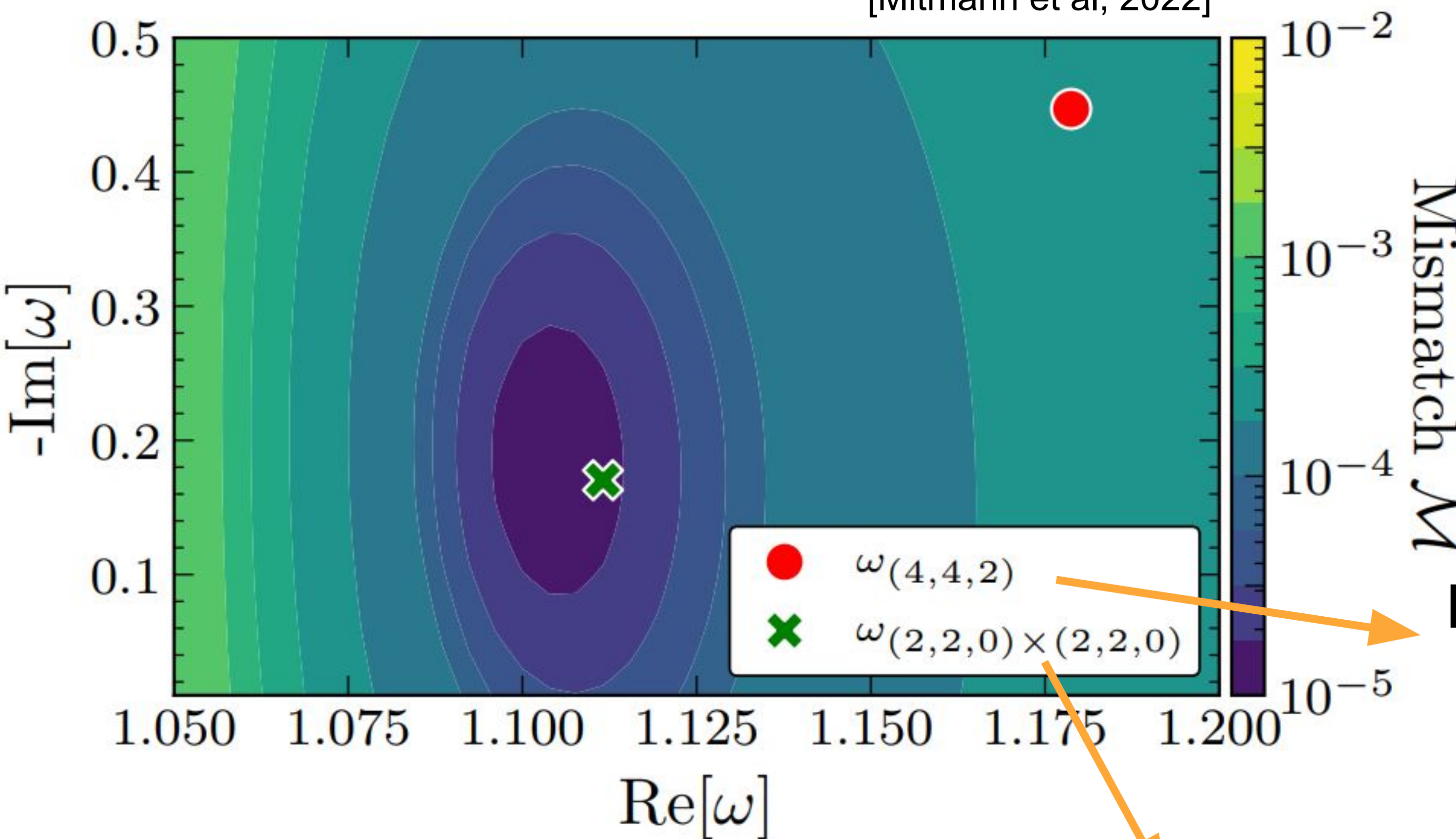
background — initial data

$$A^{(2)}_{lmn} Y_{lm} \sim \Sigma f(r; M) A^{(1)}_{lmn} A^{(1)}_{l'm'n'} \underbrace{Y_{lm} Y_{l'm'}}_{\sim G_{lm\times l'm'} Y_{lm}}$$

$$A^{(2)}_{lmn\times l'm'n'} = c_{lmn\times l'm'n'}(M, a) A^{(1)}_{lmn} A^{(1)}_{l'm'n'}$$

Non-linear model preferred @ infinity

[Mitmann et al, 2022]



Prediction
linear order

Prediction
quadratic order
in perturbation
theory

So why do I think this is exciting?

Implications for observations:

$$h^{obs} = h^{linear} + h^{non-linear}$$

but frequencies are "finger-printed" with an order in perturbation theory!

Can we also model the black hole horizon with QNMs?

Horizon should be more non-linear, but not too crazy

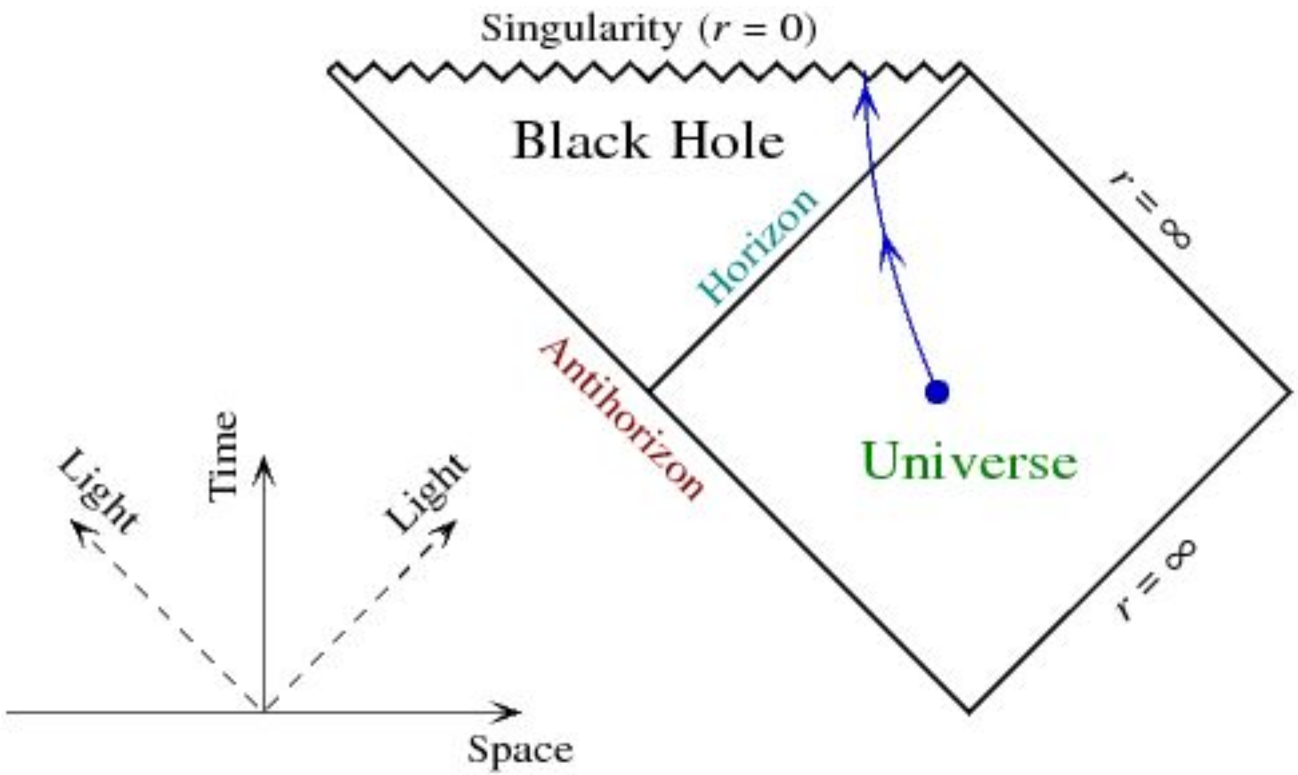
→ easier to find quadratic QNMs

Horizon is strong field regime

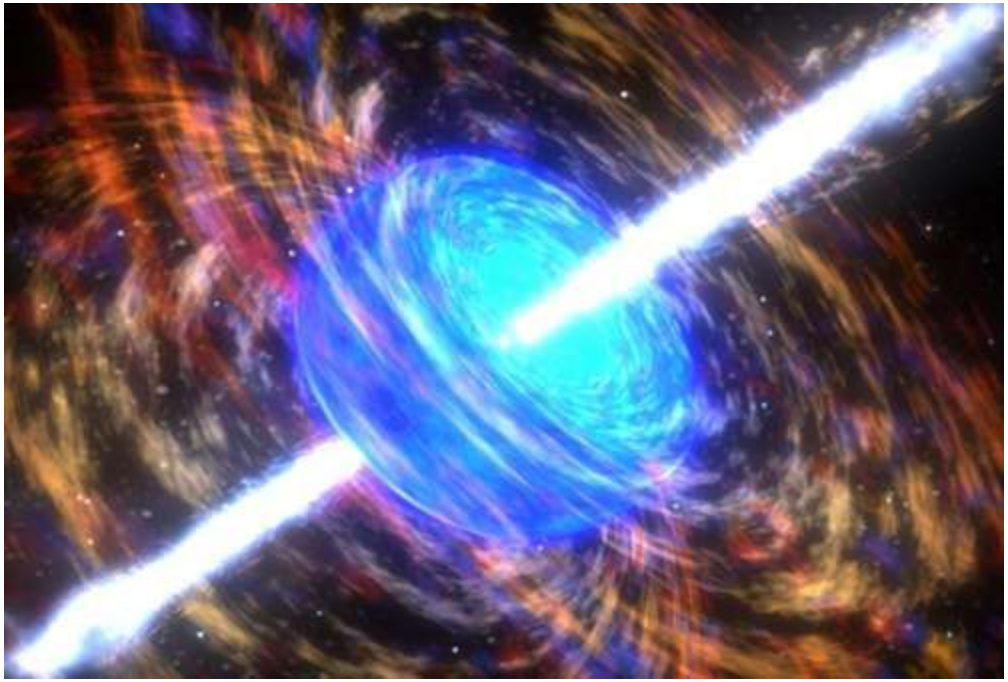
→hopeless to try to find any QNMs

Why care about the horizon…

…if observations are @ null infinity?



Electromagnetic observations and their sources



Gravitational waves…

are interesting because of their origin!

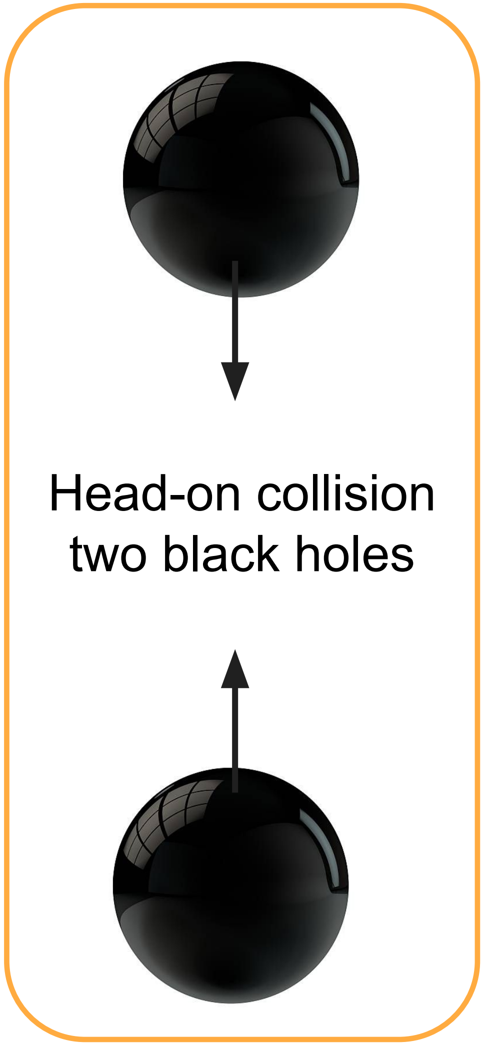
Corollary:

QNMs are interesting because they are emitted by black holes.

Disclaimer

All results are based on fitting observations.
No theoretical derivations (yet)....

Two sets of simulations using the Einstein Toolkit

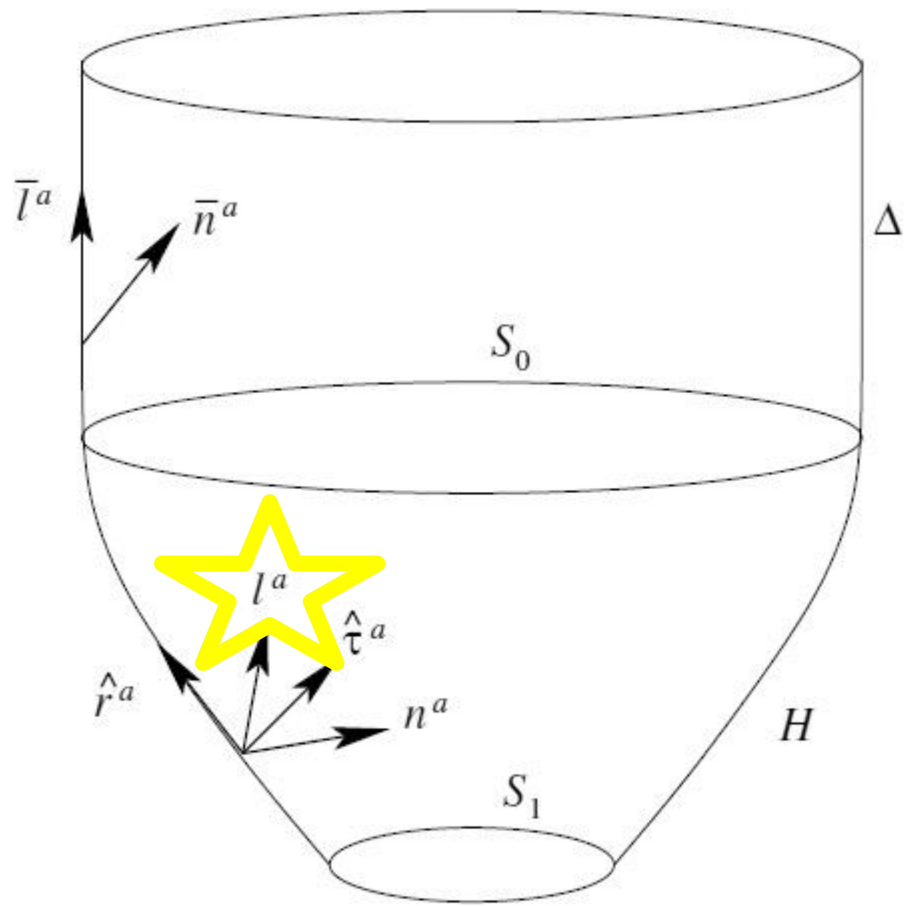


(1) Resulting BH is non-rotating
(2) Axisymmetric simulations ⟶ no m=0 modes
(3) High resolution near horizon (but poor near infinity)

- Unboosted: m1,m2,P=0
- Boosted: m1=m2, P≠ 0

⇨ linear amplitudes 10x bigger

Shear at the horizon



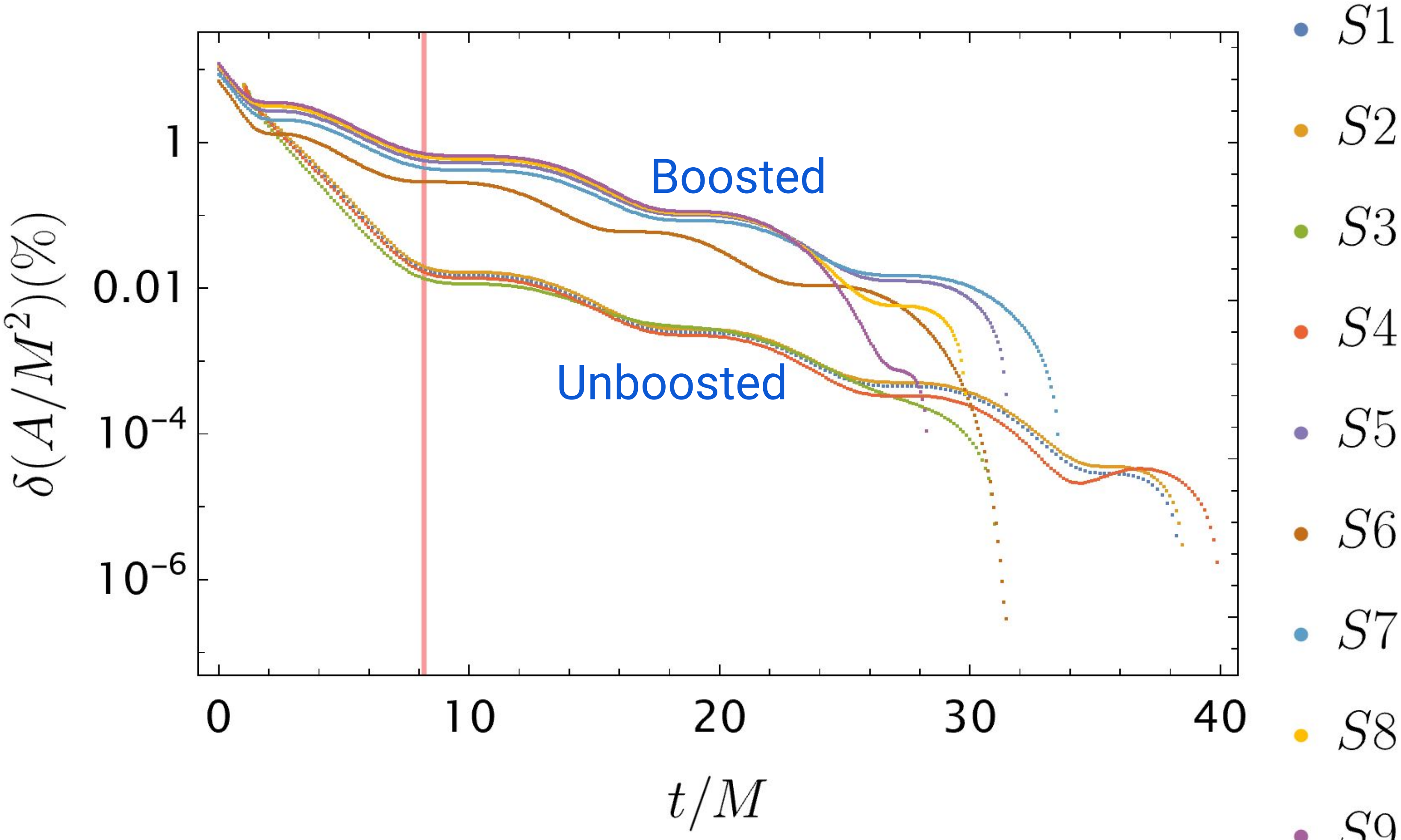
Choice of time

Time ⟷ Definition of frequency

Disclaimer: We simply use the simulation time.

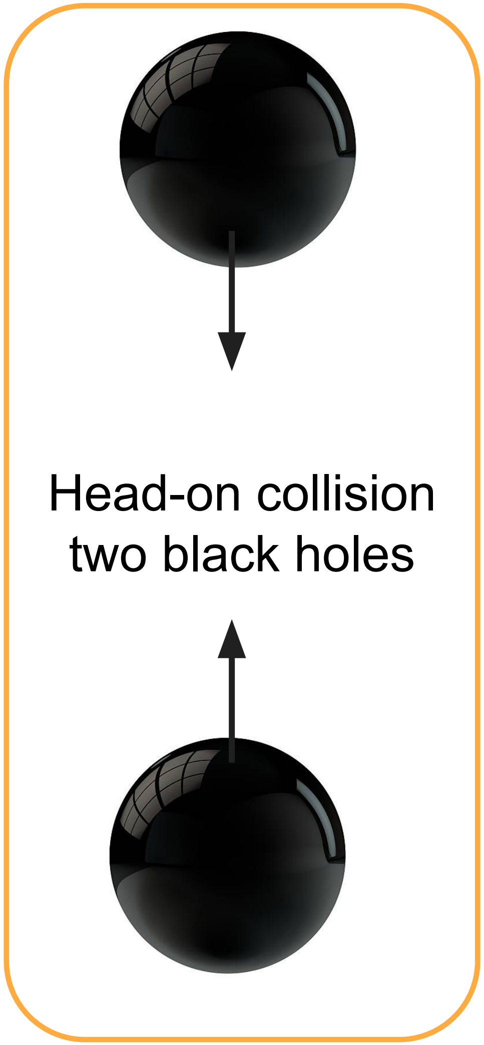
Same issue at infinity!

Ringdown: Mass changes ≤ 1 %



We take $t_{ringdown}$ = 8.2 M

Two sets of simulations using the Einstein Toolkit



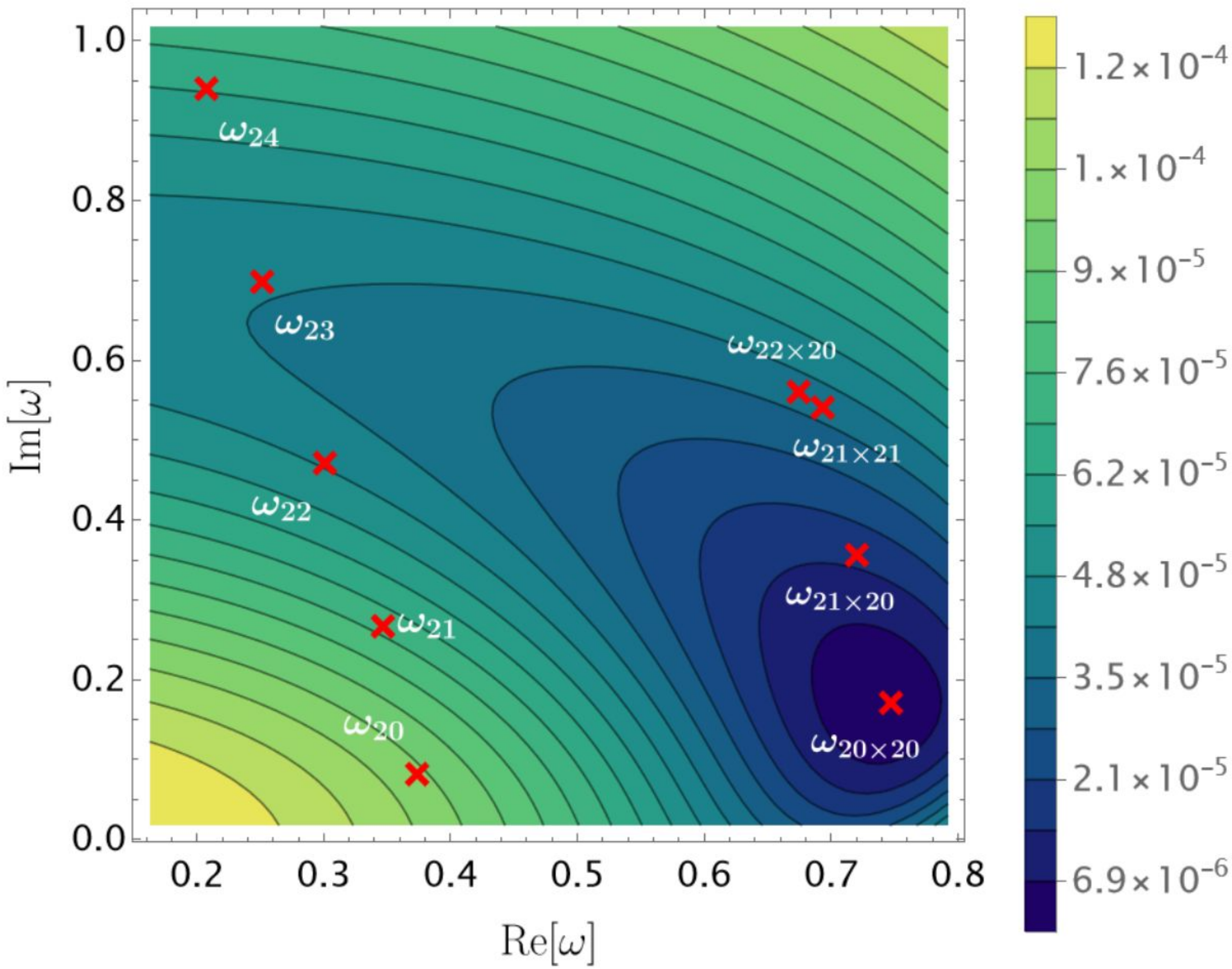
- Unboosted: m1,m2,P=0
  - ⇨ model with 4 tones
- Boosted: m1=m2, P≠ 0
  - ⇨ linear amplitudes 10x bigger
  - ⇨ model with 3 tones

S7: boosted

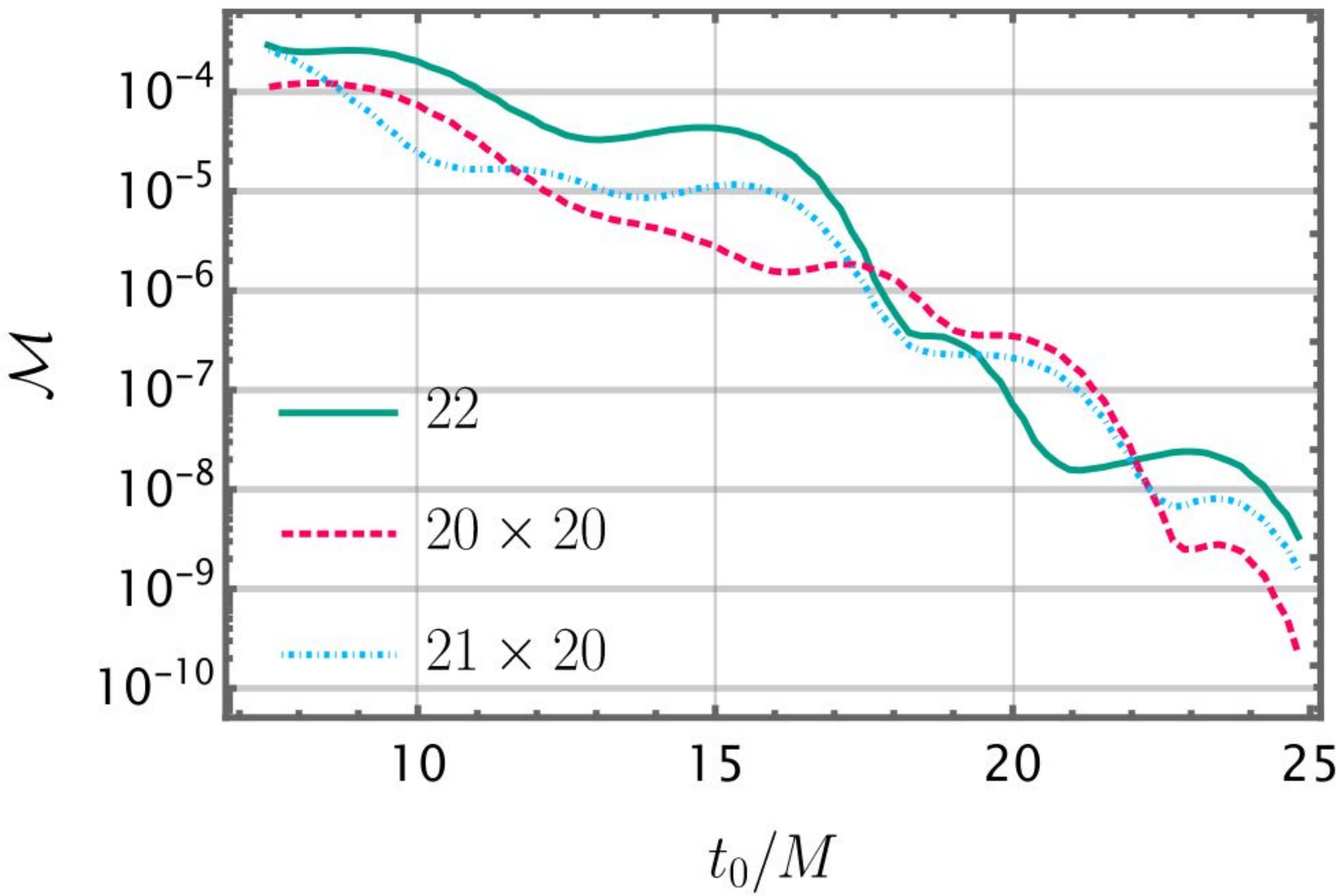
Equal mass → $l=2,4,6,\ldots$ are only non-zero.

Notation: $\omega_{lmn} \rightarrow \omega_{ln}$

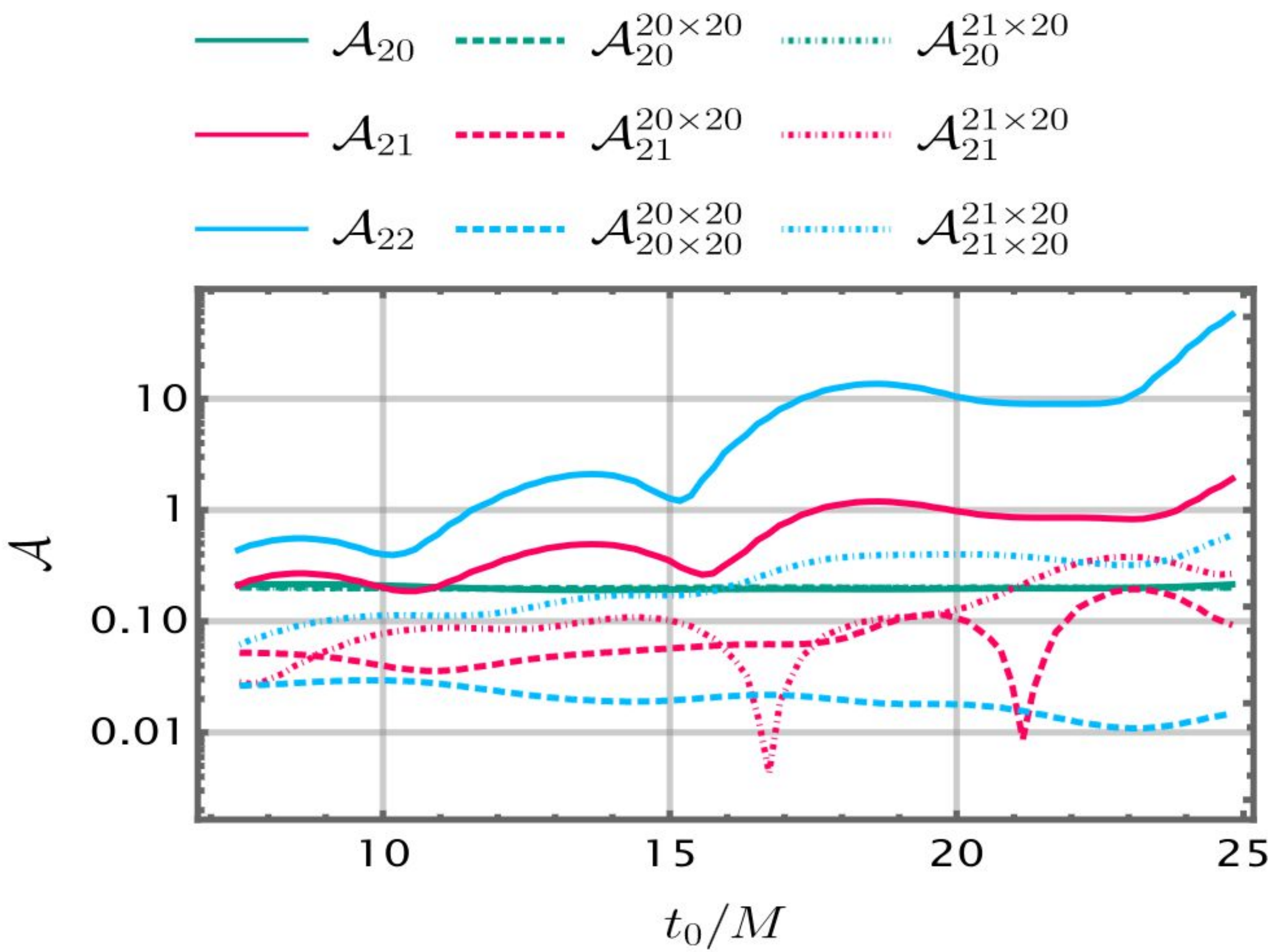
Mismatch S7 after fixing ω_{200} and ω_{201}



Mismatch S7 after fixing ω_{200} and ω_{201}

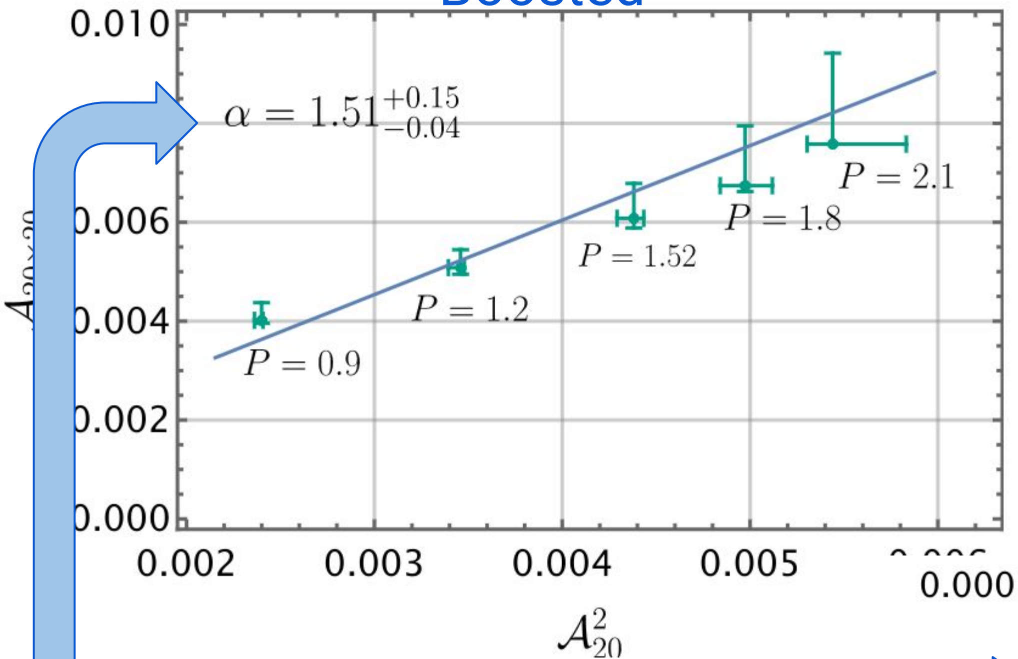


Stability amplitude

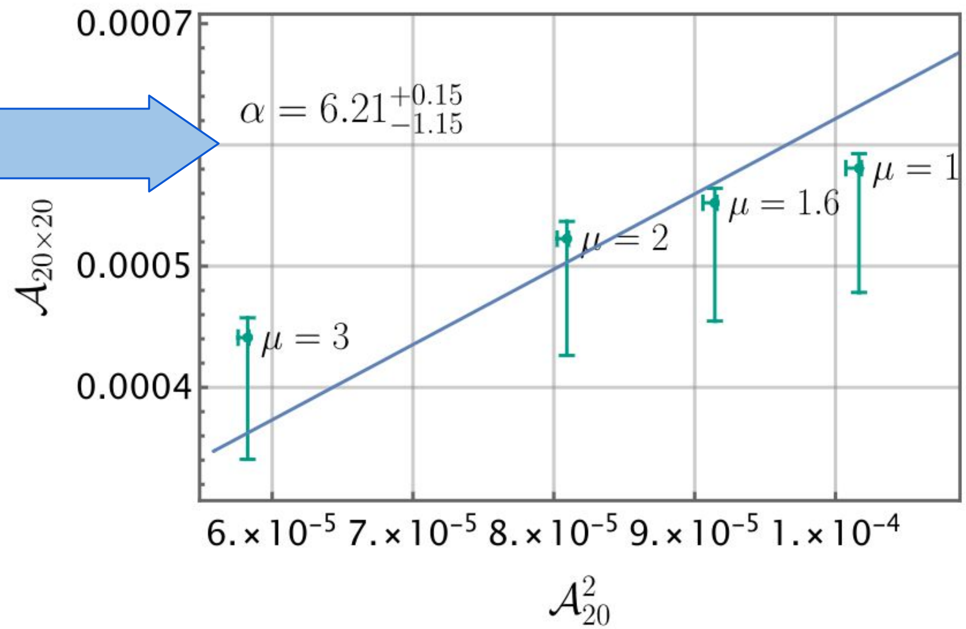


Amplitude relation

Boosted



Unboosted



Puzzle: Why are these slopes different?

Other l-modes

Mode	$\omega_{ln\times l'n'}$	Boosted (α)	Unboosted (α)
$l=2$	$\omega_{20\times 20}$	$1.51^{+0.15}_{-0.04}$	$6.21^{+0.15}_{-1.15}$
$l=4$	$\omega_{20\times 20}$	$0.73^{+0.06}_{-0.33}$	-
	$\omega_{20\times 40}$	$2.6^{+0.26}_{-0.26}$	-
$l=6$ *	$\omega_{20\times 40}$	$1.78^{0.53}_{-0.74}$	-
	$\omega_{20\times 60}$	$2.52^{+1.29}_{-0.59}$	-
	$\omega_{20\times 40}$	$1.78^{0.44}_{-0.65}$	-
	$\omega_{40\times 40}$	$2.82^{+1.5}_{-0.62}$	-

Connection horizon and infinity

- For l=4, same quadratic modes found at infinity
- For l=6, also $\omega_{200\times400}$ found at infinity

[Cheung et al, 2022 + private correspondence]

Conclusion

- ★ Quadratic QNMs fit the shear (and multipole) data at the horizon better than models with overtones
  - lower mismatch
  - more stable amplitudes wrt changes in starting time
  - closer to the optimal frequency
  - amplitude relation is satisfied

- ★ Some of the same (quadratic) modes found at horizon and infinity

- ★ Puzzling: why is the amplitude relation for boosted and unboosted simulations different?

Open questions

(1) Why are the slopes for boosted/unboosted simulations different?

(2) All results based on fitting observations, are there better ways to do this?

(3) Is there a well-motivated choice of slicing/time?

(4) Can we link observations at infinity more directly to horizon properties?